Spring 2017

6.265 - Modern Discrete Probability

Course description

Prof. G. Bresler and Y. Polyanskiy

Schedule:	MW 1-2:30p in 56-154.
Professor:	Guy Bresler <guy@mit.edu>, 32D-672
	Office hours: upon request
Professor:	Yury Polyanskiy <yp@mit.edu>, 32D-668
	Office hours: upon request
Teaching assistant:	Ilias Zadik <izadik@mit.edu>
	Office hours: TBD
Stellar:	https://stellar.mit.edu/S/course/15/sp17/15.070/
Grades:	Final assignment (30%), homeworks (60%), scribing (5%), peer grading (5%).

1 Syllabus

This is a graduate-level introduction to probabilistic reasoning for discrete systems. The topics are motivated by and applied to several foundational models: percolation, random graphs, and Ising model, as well as dynamics on them and associated computational problems. A central theme is the study of sharp thresholds.

References: lectures, supplemental reading materials, textbooks [AS04, LPW09, GS14, BLB04].

1. **Probabilistic method.** First and second moment methods. Application to percolation, broadcasting, and random graphs.

2. **Martingales.** Filtrations and stopping times. Doob maximal inequalties. Optional stopping. Application to branching processes.

3. **Concentration of measure.** Large deviations. Concentration of Lipschitz functions. Isoperimetry in the hypercube (Harper's theorem) and Gauss space. Talagrand convex distance.

4. **Mixing of Markov chains.** Glauber dynamics, Metropolis chains, MCMC. Spectral gap. Coupling and mixing times.

5. **Approximate counting.** Equivalence of sampling and counting. Computing partition functions.

6. **Analysis of Boolean functions.** Discrete Fourier decomposition. Kahn-Kalai-Linial lemma. Sharp thresholds of monotone functions.

7. **Ising models.** Gibbs measures. Broadcasting on trees. Percolation.

Homework and peer grading

You are welcome to collaborate on the homework if you like. Each student is to write up their own solutions, however, and you must write with whom you collaborated at the top of the assignment. The homework is a crucial part of the course. It is impossible to learn the material without a commitment to working the problems.

All problem sets will be peer-graded. Each problem set will contain about 4 questions, and there will be a separate grader for every question. The graders will be expected to follow these guidelines from 6.856:

`http://courses.csail.mit.edu/6.856/current/rubric.html`.

Since a separate grader will be assigned for each problem, we ask you to write your solution to each problem on its own page. Write your name on each problem separately, in order to have your solution graded.

Late Homework Policy

Late submissions up to 2 days will be penalized 20% **per day**. No credit will be given if the submission is late by more than 2 days. Coordinate with the graders directly to turn in late assignments.

Exams

There will be a final take-home assignment. There are no mid-term exams.

References

[AS04] Noga Alon and Joel H Spencer. *The probabilistic method*. John Wiley & Sons, 2004.

[BLB04] S. Boucheron, G. Lugosi, and O. Bousquet. Concentration inequalities. In O. Bousquet, U. von Luxburg, and G. Rätsch, editors, *Advanced Lectures on Machine Learning*, pages 208–240. Springer, 2004.

[GS14] Christophe Garban and Jeffrey E Steif. *Noise sensitivity of Boolean functions and percolation*, volume 5. Cambridge University Press, 2014.

[LPW09] David Asher Levin, Yuval Peres, and Elizabeth Lee Wilmer. *Markov chains and mixing times*. American Mathematical Soc., 2009.